



**CERTIFICATES OF COMPETENCY IN THE MERCHANT NAVY
MARINE ENGINEER OFFICER**

**EXAMINATIONS ADMINISTERED BY THE
SCOTTISH QUALIFICATIONS AUTHORITY
ON BEHALF OF THE
MARITIME AND COASTGUARD AGENCY**

STCW 95 MANAGEMENT ENGINEER REG. III/2 (UNLIMITED)

**Applied Heat
December 2018 Solution**

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Note:-

This solution is for private circulation only. Not for sale.

Every effort has been made to see that there are no errors (typographical or otherwise) in the material presented. However, it is still possible that there are a few errors (serious or otherwise). We would be thankful to the reader, if they are brought to my attention at the following e-mail address: ulheyogesh@gmail.com



Q.1

Air at a pressure and temperature of 5.5 bar and 1300 K respectively is cooled at constant volume until the pressure is 1.35 bar.

The air is then reversibly compressed according to the law $pV^{1.28} = \text{constant}$ back to the original pressure.

(a) Sketch the sequence of process on pressure-Volume and Temperature-specific entropy diagrams. (2)

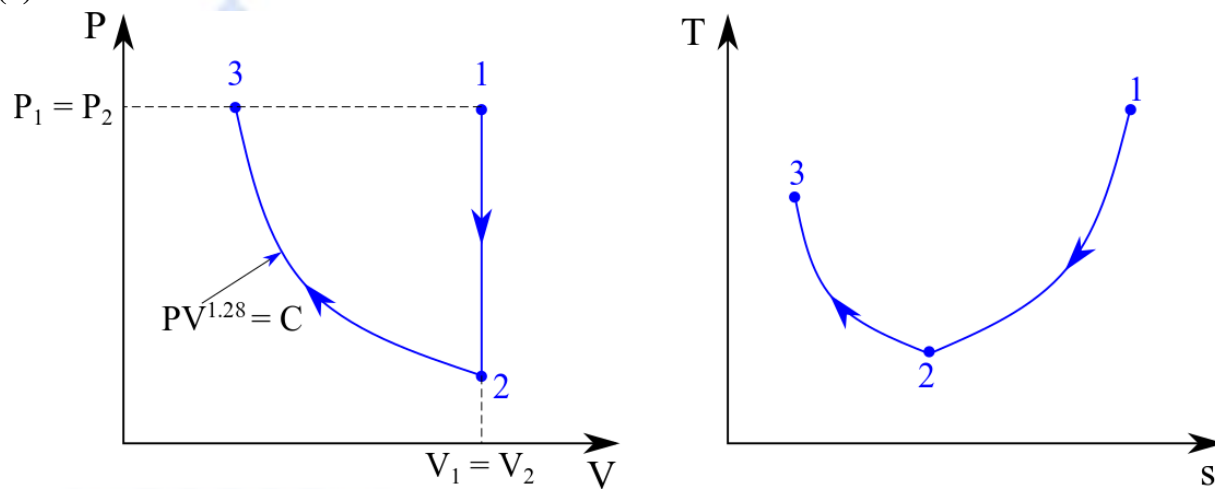
(b) Calculate EACH of the following for 1 kg of air:

- (i) the work transfer; (4)
- (ii) the total change in internal energy; (3)
- (iii) the net heat transfer; (4)
- (iv) the overall change in entropy. (3)

Note: for air $R = 0.287 \text{ kJ/kgK}$ and $c_v = 0.718 \text{ kJ/kgK}$

Solution:

(a)



(b)

$$P_1 = 5.5 \text{ bar} = 550 \text{ kN/m}^2, \quad T_1 = 1300 \text{ K}, \quad V_1 = V_2,$$

$$P_2 = 1.35 \text{ bar} = 135 \text{ kN/m}^2, \quad P_3 = P_1 = 550 \text{ kN/m}^2$$

$$R = 0.287 \text{ kJ/kgK}, \quad c_v = 0.718 \text{ kJ/kgK}$$

$$c_p = R + c_v = 0.287 + 0.718 = 1.005 \text{ kJ/kgK}$$

For process 1-2

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \Rightarrow \quad T_2 = \frac{P_2}{P_1} \times T_1 = \frac{135}{550} \times 1300 = 319.09 \text{ K}$$

P.T.O.

For process 2-3

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} \Rightarrow T_3 = T_2 \left(\frac{P_3}{P_2}\right)^{\frac{n-1}{n}} = 319.09 \times \left(\frac{550}{135}\right)^{\frac{0.28}{1.28}} = 433.867 \text{ K}$$

(b)(i)

$$W_{1-2} = 0 \text{ kJ/kg}$$

$$W_{2-3} = \frac{mR(T_2 - T_3)}{n - 1} = \frac{1 \times 0.287 \times (319.09 - 433.867)}{0.28} = -117.64 \text{ kJ/kg}$$

$$W_{Total} = W_{1-2} + W_{2-3} = 0 + (-117.64) = -117.64 \text{ kJ/kg}$$

$$\Rightarrow W_{Total} = 117.64 \text{ kJ/kg (work input)}$$

Therefore, the work transfer is 117.64 kJ/kg (work input).

(b)(ii)

$$\Delta U_{1-2} = mc_v(T_2 - T_1) = 1 \times 0.718 \times (319.09 - 1300) = -704.293 \text{ kJ/kg}$$

$$\Delta U_{2-3} = mc_v(T_3 - T_2) = 1 \times 0.718 \times (433.867 - 319.09) = 82.41 \text{ kJ/kg}$$

$$\Delta U_{Total} = \Delta U_{1-2} + \Delta U_{2-3} = -704.293 + 82.41 = -621.883 \text{ kJ/kg}$$

Therefore, the total change in internal energy is 621.883 kJ/kg (reduction).

(b)(iii)

$$Q_{1-2} = mc_v(T_2 - T_1) = -704.293 \text{ kJ/kg}$$

$$Q_{2-3} = \Delta U_{2-3} + W_{2-3} = 82.41 + (-117.64) = -35.23 \text{ kJ/kg}$$

$$Q_{Total} = Q_{1-2} + Q_{2-3} = -704.293 + (-35.23) = -739.523 \text{ kJ/kg}$$

$$\Rightarrow Q_{Total} = 739.523 \text{ kJ/kg (heat rejection)}$$

Therefore, the net heat transfer is 739.523 kJ/kg (heat rejection).

(b)(iv)

$$\Delta s_{1-2} = mc_v \ln\left(\frac{T_2}{T_1}\right) = 1 \times 0.718 \times \ln\left(\frac{319.09}{1300}\right) = -1.0085 \text{ kJ/kgK}$$

$$\Delta s_{2-3} = m \left[c_p \ln\left(\frac{T_3}{T_2}\right) - R \ln\left(\frac{P_3}{P_2}\right) \right] = 1 \left[1.005 \times \ln\left(\frac{433.867}{319.09}\right) - 0.287 \times \ln\left(\frac{550}{135}\right) \right]$$

$$\Rightarrow \Delta s_{2-3} = -0.09433 \text{ kJ/kgK}$$

$$\Delta s_{Total} = \Delta s_{1-2} + \Delta s_{2-3} = -1.0085 + (-0.09433) = -1.10283 \text{ kJ/kgK}$$

Therefore, the overall change in entropy is 1.10283 kJ/kgK (reduction).

Q.2

In an air standard dual combustion cycle the volume compression ratio is 20:1.

The minimum pressure and temperature are 2.0 bar and 47°C respectively.

The maximum pressure is 200 bar and the maximum temperature is 1685°C.

(a) Sketch the cycle on pressure-Volume and Temperature-specific entropy diagrams. (3)

(b) Calculate EACH of the following:

(i) the pressure and temperature at each point in the cycle; (5)

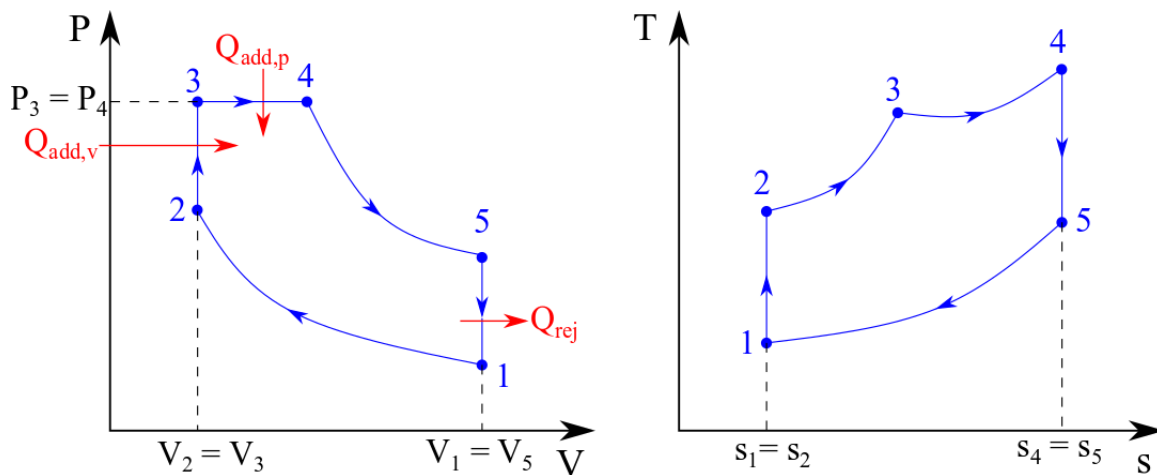
(ii) the percentage of the total heat added at constant volume; (5)

(iii) the cycle thermal efficiency. (3)

Note: for air $\gamma = 1.4$ and $R = 0.287$ kJ/kgK

Solution:

(a)



(b)

$$r = \frac{V_1}{V_2} = 20$$

$$P_{min} = P_1 = 2 \text{ bar} = 200 \text{ kN/m}^2$$

$$T_{min} = T_1 = 47^\circ\text{C} = 320 \text{ K}$$

$$P_{max} = P_3 = P_4 = 200 \text{ bar} = 20000 \text{ kN/m}^2$$

$$T_{max} = T_4 = 1685^\circ\text{C} = 1958 \text{ K}$$

$$\gamma = 1.4, \quad R = 0.287 \text{ kJ/kgK}$$

$$c_v = \frac{R}{\gamma - 1} = \frac{0.287}{0.4} = 0.7175 \text{ kJ/kgK}$$

$$c_p = \gamma c_v = 1.4 \times 0.7175 = 1.0045 \text{ kJ/kgK}$$

P.T.O.

(b)(i)

$$T_2 = T_1 r^{\gamma-1} = 320 \times 20^{0.4} = \mathbf{1060.625 K}$$

For process 1-2

$$P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow P_2 = P_1 \times \left(\frac{V_1}{V_2}\right)^\gamma = 200 \times 20^{1.4} = \mathbf{13257.82 kN/m^2}$$

For process 2-3

$$\frac{P_2}{T_2} = \frac{P_3}{T_3} \Rightarrow T_3 = \frac{P_3}{P_2} \times T_2 = \frac{20000}{13257.82} \times 1060.625 = \mathbf{1600 K}$$

For process 3-4

$$\frac{V_3}{T_3} = \frac{V_4}{T_4} \Rightarrow \frac{V_4}{V_3} = \frac{T_4}{T_3} = \frac{1958}{1600} = 1.22375$$

$$\therefore r_c = \frac{V_4}{V_3} = 1.22375$$

For process 4-5

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma-1} = \left(\frac{V_4}{V_3} \times \frac{V_3}{V_5}\right)^{\gamma-1} = \left(r_c \times \frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{r_c}{r}\right)^{\gamma-1}$$

$$\Rightarrow T_5 = T_4 \left(\frac{r_c}{r}\right)^{\gamma-1} = 1958 \times \left(\frac{1.22375}{20}\right)^{0.4} = \mathbf{640.44 K}$$

For process 4-5

$$\frac{T_5}{T_4} = \left(\frac{P_5}{P_4}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow P_5 = P_4 \left(\frac{T_5}{T_4}\right)^{\frac{\gamma}{\gamma-1}} = 20000 \times \left(\frac{640.44}{1958}\right)^{\frac{1.4}{0.4}} = \mathbf{400.276 kN/m^2}$$

Therefore, the pressure and temperature at each point in the cycle are

$T_1 = 320 K$	$P_1 = 200 kN/m^2$
$T_2 = 1060.625 K$	$P_2 = 13257.82 kN/m^2$
$T_3 = 1600 K$	$P_3 = 20000 kN/m^2$
$T_4 = 1958 K$	$P_4 = 20000 kN/m^2$
$T_5 = 640.44 K$	$P_5 = 400.276 kN/m^2$

(b)(ii)

$$Q_{add,v} = c_v(T_3 - T_2) = 0.7175 \times (1600 - 1060.625) = 387.001 \text{ kJ/kg}$$

$$Q_{add,p} = c_p(T_4 - T_3) = 1.0045 \times (1958 - 1600) = 359.611 \text{ kJ/kg}$$

$$Q_{add,total} = Q_{add,v} + Q_{add,p} = 387.001 + 359.611 = 746.612 \text{ kJ/kg}$$

$$\begin{aligned} \% \text{ total heat added at constant volume} &= \frac{Q_{add,v}}{Q_{add,total}} \times 100 = \frac{387.001}{746.612} \times 100 \\ &= 51.834\% \end{aligned}$$

Therefore, the percentage of total heat added at constant volume is 51.834%.

(b)(iii)

$$Q_{rej} = c_v(T_5 - T_1) = 0.7175 \times (640.44 - 320) = 229.9157 \text{ kJ/kg}$$

$$\eta_{th} = 1 - \frac{Q_{rej}}{Q_{add,total}} = 1 - \frac{229.9157}{746.612} = 0.692 = 69.2\%$$

Therefore, the cycle thermal efficiency is 69.2%.

Q.5

Steam, at a pressure and temperature of 4 bar and 200°C respectively, leaves the fixed blades of a 50% reaction turbine stage.

The moving blade inlet and outlet angles are 50° and 35° respectively.

The mean blade speed is 170 m/s.

The blade height is 10% of the blade ring mean diameter.

The mass flow rate of steam through the stage is 14 kg/s.

Calculate EACH of the following:

- the blade height; (5)
- the stage power; (4)
- the percentage increase in the moving blade relative velocity; (4)
- the stage specific enthalpy drop. (3)

Solution:

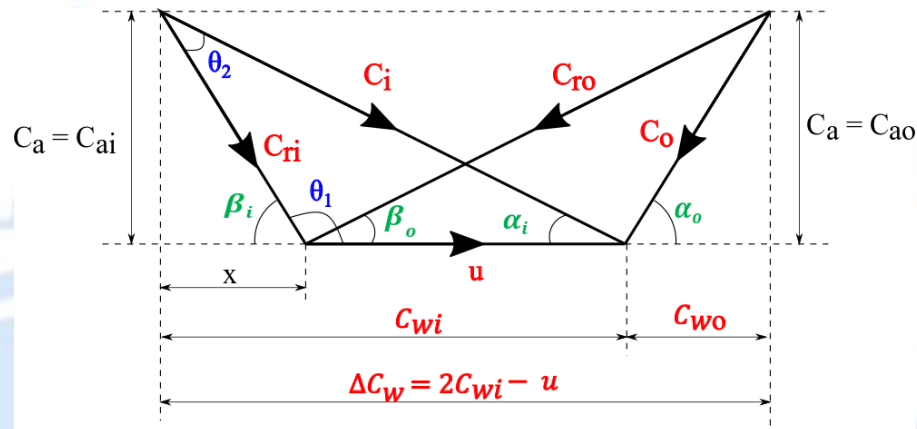


Fig:- Combined velocity diagram

$$P = 4 \text{ bar}, \quad T = 200^\circ \text{C}$$

$$\beta_i = 50^\circ, \quad \beta_o = 35^\circ, \quad u = 170 \text{ m/s}$$

$$h = 10\%D = 0.1D, \quad \dot{m} = 14 \text{ kg/s}$$

$$v = v@4\text{bar and } 200^\circ \text{C} = 0.5345 \text{ m}^3/\text{kg}$$

Due to symmetry in combined velocity diagram,

$$\alpha_i = \beta_o = 35^\circ, \quad \alpha_o = \beta_i = 50^\circ$$

$$\theta_1 = 180^\circ - \beta_i = 180^\circ - 50^\circ = 130^\circ$$

$$\theta_2 = 180^\circ - \theta_1 - \alpha_i = 180^\circ - 130^\circ - 35^\circ = 15^\circ$$

By sine rule,

$$\frac{C_i}{\sin \theta_1} = \frac{u}{\sin \theta_2}$$

$$\Rightarrow C_i = \sin \theta_1 \times \frac{u}{\sin \theta_2} = \sin 130 \times \frac{170}{\sin 15} = 503.16 \text{ m/s}$$

$$C_a = C_i \sin \alpha_i = 503.16 \times \sin 35 = 288.6 \text{ m/s}$$

$$C_{wi} = C_i \cos \alpha_i = 503.16 \times \cos 35 = 412.164 \text{ m/s}$$

$$\dot{m} = \frac{\pi D h C_a}{v} = \frac{\pi D \times 0.1 D \times C_a}{v} \quad \{\because h = 0.1 D\}$$

$$\Rightarrow \dot{m} = \frac{\pi \times 0.1 D^2 C_a}{v}$$

$$\Rightarrow D = \sqrt{\frac{\dot{m} v}{0.1 \pi C_a}} = \sqrt{\frac{14 \times 0.5345}{0.1 \times \pi \times 288.6}} = 0.2873 \text{ m}$$

(a)

$$h = 0.1 D = 0.1 \times 0.2873 = 0.02873 \text{ m} = 28.73 \text{ mm}$$

Therefore, the blade height is 28.73 mm.

(b)

$$\text{Stage power} = \dot{m} \times u \times \Delta C_w = \dot{m} \times u \times (2C_{wi} - u)$$

$$\text{Stage power} = 14 \times 170 \times (2 \times 412.164 - 170) = 1557.3 \times 10^3 = 1557.3 \text{ kW}$$

Therefore, the stage power is 1557.3 kW.

(c)

Due to symmetry in velocity diagram,

$$C_{ro} = C_i = 503.16 \text{ m/s}$$

$$x = C_{wi} - u = 412.164 - 170 = 242.164 \text{ m/s}$$

$$C_{ri} = \sqrt{x^2 + C_a^2} = \sqrt{242.164^2 + 288.6^2} = 376.74 \text{ m/s}$$

$$\% \text{ increase in moving blade relative velocity} = \frac{C_{ro} - C_{ri}}{C_{ri}} \times 100$$

$$\Rightarrow \% \text{ increase in moving blade relative velocity} = \frac{503.16 - 376.74}{376.74} \times 100$$

$$\Rightarrow \% \text{ increase in moving blade relative velocity} = 33.56\%$$

Therefore, the percentage increase in moving blade relative velocity is 33.56%.

(d)

Due to symmetry in combined velocity diagram,

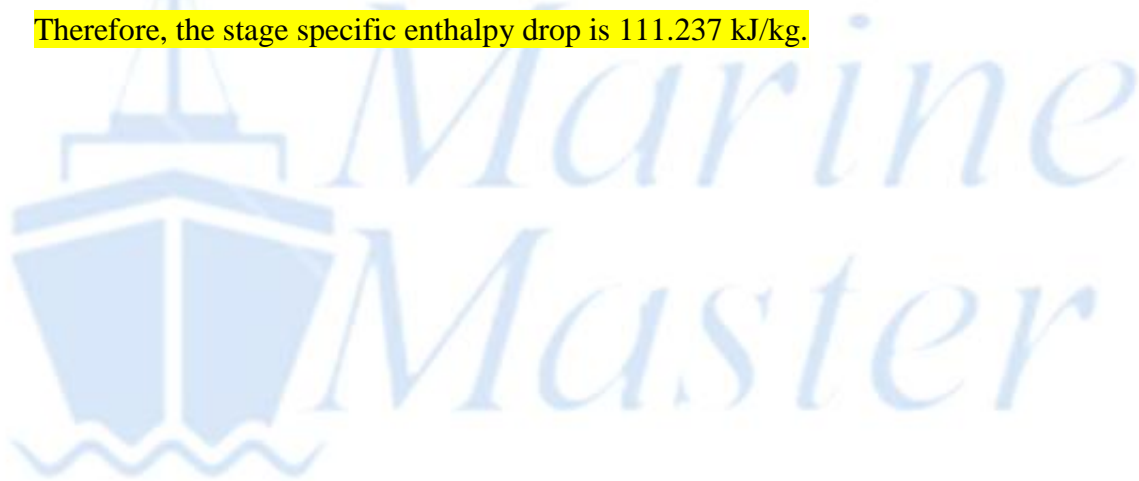
$$C_o = C_{ri} = 376.74 \text{ m/s}$$

$$\text{Stage specific enthalpy drop} = \frac{C_{ro}^2 - C_{ri}^2}{2} + \frac{C_i^2 - C_o^2}{2}$$

$$\Rightarrow \text{Stage specific enthalpy drop} = \frac{503.16^2 - 376.74^2}{2} + \frac{503.16^2 - 376.74^2}{2}$$

$$\Rightarrow \text{Stage specific enthalpy drop} = 111.237 \times 10^3 \text{ J/kg} = 111.237 \text{ kJ/kg}$$

Therefore, the stage specific enthalpy drop is 111.237 kJ/kg.



Q.6

A vapour compression refrigeration plant uses R134a and operates between pressures of 1.0637 bar and 10.163 bar.

The refrigerant enters the compressor at a temperature of -25°C and leaves at a temperature of 55°C .

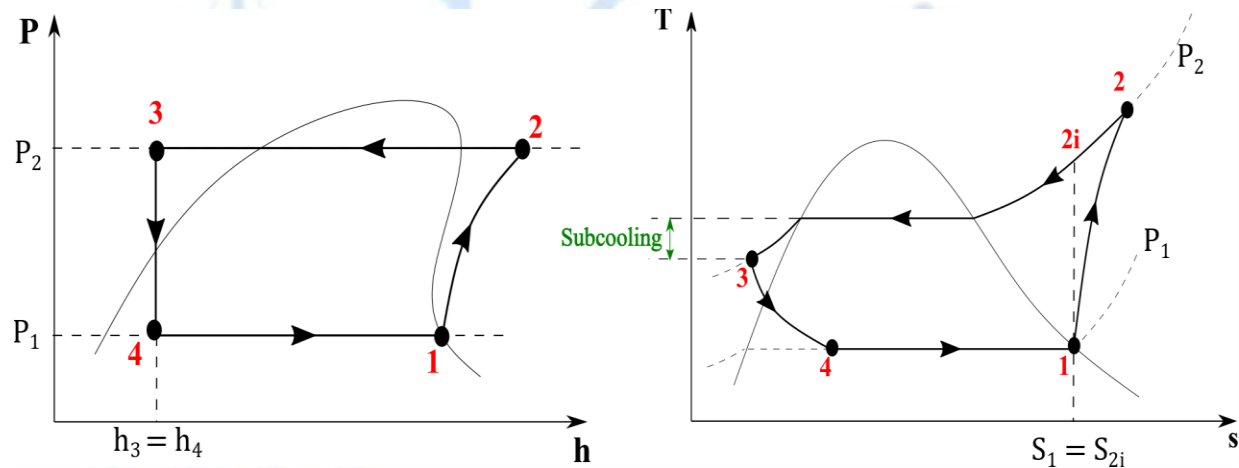
The refrigerant leaves the condenser with 10 K of subcooling.

At these conditions the power input to the plant is 117 kW with a mechanical efficiency of 90%.

- (a) Sketch the cycle on pressure-specific enthalpy and Temperature-specific entropy diagrams. (4)
- (b) Calculate EACH of the following:
- the isentropic efficiency of the compressor; (6)
 - the cooling load; (4)
 - the plant coefficient of performance. (2)

Solution:

(a)



(b)

Refrigerant is R134a

$$P_1 = 1.0637 \text{ bar}, \quad P_2 = 10.163 \text{ bar}$$

$$T_1 = -25^{\circ}\text{C}, \quad T_2 = 55^{\circ}\text{C}$$

$$\text{Subcooling in condenser} = 10 \text{ K}$$

$$\text{Power input} = 117 \text{ kW}, \quad \eta_{\text{mech}} = 90\% = 0.9$$

$$T_{\text{sat}@P_1} = -25^{\circ}\text{C}, \quad T_{\text{sat}@P_2} = 40^{\circ}\text{C}$$

P.T.O.

As $T_1 = T_{sat}@P_1$, it can be assumed that refrigerant enters the compressor as saturated vapour.

$$h_1 = h_g@P_1 = 383.37 \text{ kJ/kg}$$

$$s_{2i} = s_1 = s_g@P_1 = 1.7457 \text{ kJ/kgK}$$

$$\text{Superheat}_2 = T_2 - T_{sat}@P_2 = 55 - 40 = 15 \text{ K}$$

$$h_2 = h@P_2 \text{ and } 15 \text{ K superheat} = \frac{430.55 + 441.32}{2} = 435.935 \text{ kJ/kg}$$

At $P_2 = 10.163 \text{ bar}$

s (kJ/kgK)	h (kJ/kg)
1.7109	419.41
$s_{2i} = 1.7457$	h_{2i}
1.7460	430.55

By linear interpolation,

$$h_{2i} = 419.41 + (1.7457 - 1.7109) \times \frac{430.55 - 419.41}{1.7460 - 1.7109} = 430.4547 \text{ kJ/kg}$$

$$T_3 = T_{sat}@P_2 - 10 = 40 - 10 = 30^\circ\text{C}$$

$$h_4 = h_3 = h_f@T_3 = 241.69 \text{ kJ/kg}$$

(b)(i)

$$\eta_c = \frac{h_{2i} - h_1}{h_2 - h_1} = \frac{430.4547 - 383.37}{435.935 - 383.37} = 0.8957 = 89.57\%$$

Therefore, the isentropic efficiency of the compressor is 89.57%.

(b)(ii)

$$\text{Power input} = \frac{\dot{m}(h_2 - h_1)}{\eta_{mech}}$$

$$\Rightarrow \dot{m} = \frac{\text{Power input} \times \eta_{mech}}{(h_2 - h_1)} = \frac{117 \times 0.9}{435.935 - 383.37} = 2.0032 \text{ kg/s}$$

$$\text{Cooling load} = \dot{m}(h_1 - h_4) = 2.0032 \times (383.37 - 241.69) = 283.81 \text{ kW}$$

Therefore, the cooling load is 283.81 kW.

(b)(iii)

$$COP = \frac{\text{Cooling load}}{\text{Power input}} = \frac{283.81}{117} = 2.425$$

Therefore, the plant coefficient of performance is 2.425.



Q.7

An air cooled heat exchanger has 9 tubes each 40 mm mean diameter in a single pass, parallel flow arrangement.

Fresh water flows through the tubes with a velocity of 0.2 m/s. It enters at a temperature of 85°C, and leaves at a temperature of 75°C.

The air enters the cooler at a temperature of 4°C and has a mass flow of 9 kg/s.

(a) Calculate EACH of the following:

- (i) the rate of heat transfer from the water; (3)
- (ii) the log mean temperature difference for the cooler; (5)
- (iii) the length of the EACH cooler tube. (3)

(b) Sketch the cooler temperature distribution (profile) diagram. (3)

Note: for air $c_p=1.005$ kJ/kgK

for water $c=4.2$ kJ/kgK

heat transfer coefficient for air side of the tube = 8.84 kW/m²K

heat transfer coefficient for water side of the tube = 13.74 kW/m²K

Solution:

(a)

$$\text{Number of tubes} = 9$$

$$D = 40 \text{ mm} = 0.04 \text{ m}$$

Hot fluid is fresh water.

$$\therefore \rho_{\text{water}} = 1000 \text{ kg/m}^3$$

$$\text{Velocity of water} = V_{\text{water}} = 0.2 \text{ m/s}$$

$$T_{H1} = 85^\circ\text{C}, T_{H2} = 75^\circ\text{C} \text{ (Water inlet and outlet temperature)}$$

Cold fluid is air.

$$T_{C1} = 4^\circ\text{C} \text{ (Air inlet temperature)}$$

$$\dot{m}_{\text{air}} = 9 \text{ kg/s}$$

$$c_{p \text{ air}} = 1.005 \text{ kJ/kgK}, \quad c_{\text{water}} = 4.2 \text{ kJ/kgK}$$

$$h_{\text{outer}} = 8.84 \text{ kW/m}^2\text{K} = 8.84 \times 10^3 \text{ W/m}^2\text{K}$$

$$h_{\text{inner}} = 13.74 \text{ kW/m}^2\text{K} = 13.74 \times 10^3 \text{ W/m}^2\text{K}$$

$$\dot{m}_{\text{water}} = \rho_{\text{water}} \times A_{\text{tube}} \times V_{\text{water}} = 1000 \times 9 \times \frac{\pi}{4} \times 0.04^2 \times 0.2 = 2.2619 \text{ kg/s}$$

(a)(i)

$$\dot{Q} = \dot{m}_{\text{water}} \times c_{\text{water}} \times (T_{H1} - T_{H2}) = 2.2619 \times 4.2 \times (85 - 75)$$

$$\Rightarrow \dot{Q} = 95 \text{ kW} = 95 \times 10^3 \text{ W}$$

Therefore, the rate of heat transfer from the water is 95 kW.

(a)(ii)

$$\dot{Q} = \dot{m}_{air} \times c_{p\ air} \times (T_{C2} - T_{C1}) \text{ in KW}$$

$$\Rightarrow T_{C2} = \frac{\dot{Q}}{\dot{m}_{air} \times c_{p\ air}} + T_{C1} = \frac{95}{9 \times 1.005} + 4 = 14.503^\circ\text{C}$$

$$\theta_1 = T_{H1} - T_{C1} = 85 - 4 = 81^\circ\text{C}$$

$$\theta_2 = T_{H2} - T_{C2} = 75 - 14.503 = 60.497^\circ\text{C}$$

$$LMTD = \frac{\theta_1 - \theta_2}{\ln\left(\frac{\theta_1}{\theta_2}\right)} = \frac{81 - 60.497}{\ln\left(\frac{81}{60.497}\right)} = 70.2505^\circ\text{C}$$

Therefore, the log mean temperature difference for the cooler is 70.2505°C .

(a)(iii)

$$\dot{Q} = \frac{LMTD}{R_{th}}$$

$$\Rightarrow R_{th} = \frac{LMTD}{\dot{Q}} = \frac{70.2505}{95 \times 10^3} = 7.3947 \times 10^{-4} \text{ K/W}$$

$$R_{th} = R_{inner} + R_{outer}$$

$$\Rightarrow R_{th} = \frac{1}{h_{inner} \times A_{inner}} + \frac{1}{h_{outer} \times A_{outer}}$$

$$\Rightarrow R_{th} = \frac{1}{h_{inner} \times 9 \times \pi D l} + \frac{1}{h_{outer} \times 9 \times \pi D l} = \left(\frac{1}{h_{inner}} + \frac{1}{h_{outer}}\right) \times \frac{1}{9 \times \pi D l}$$

$$\Rightarrow 7.3947 \times 10^{-4} = \left(\frac{1}{13.74 \times 10^3} + \frac{1}{8.84 \times 10^3}\right) \times \frac{1}{9 \times \pi \times 0.04 \times l}$$

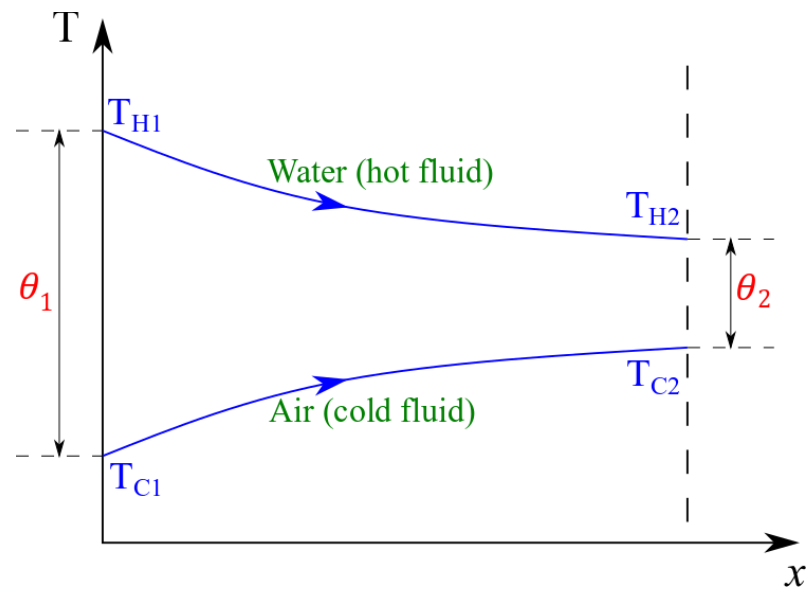
$$\Rightarrow 7.3947 \times 10^{-4} = \frac{1.6437 \times 10^{-4}}{l}$$

$$\Rightarrow l = \frac{1.6437 \times 10^{-4}}{7.3947 \times 10^{-4}} = 0.2223 \text{ m} = 222.3 \text{ mm}$$

Therefore, the length of the each cooler tube is 222.3 mm.

P.T.O.

(b)



Q.8

A single acting, three stage reciprocating air compressor, is designed for minimum work with perfect intercooling.

It delivers 8 kg/min of air from initial conditions of 1.15 bar and 25°C and has a volumetric efficiency of 0.88 at a speed of 360 rev/min.

The clearance volume in each stage is 5% of the respective swept volume.

Compression and expansion processes take place according to the law $pV^{1.25} = \text{constant}$.

(a) Calculate EACH of the following:

- (i) the stage delivery pressures; (5)
- (ii) the indicated power; (3)
- (iii) the total heat removed in intercoolers. (4)

(b) Sketch the cycle on a pressure-Volume diagram, indicating the stage pressures.

(4)

Note: for air $R = 287 \text{ J/kgK}$, $c_p = 1005 \text{ J/kgK}$

Solution:

(a)

$$\text{Number of stages} = 3$$

$$\dot{m} = 8 \text{ kg/min} = \frac{8}{60} \text{ kg/s} = 0.1333 \text{ kg/s}$$

$$P_1 = 1.15 \text{ bar} = 115 \text{ kN/m}^2$$

$$T_1 = 25^\circ\text{C} = 298 \text{ K}$$

$$\eta_{vol} = 0.88$$

$$N = 360 \text{ rev/min}$$

$$V_c = 5\%V_s = 0.05V_s \Rightarrow \frac{V_c}{V_s} = 0.05$$

$$n = 1.25$$

$$R = 287 \text{ J/kgK} = 0.287 \text{ kJ/kgK}$$

$$c_p = 1005 \text{ J/kgK} = 1.005 \text{ kJ/kgK}$$

(a)(i)

$$\eta_{vol} = 1 + \frac{V_c}{V_s} \left[1 - \left(\frac{P_2}{P_1} \right)^{1/n} \right]$$

$$\Rightarrow 0.88 = 1 + 0.05 \times \left[1 - \left(\frac{P_2}{P_1} \right)^{1/n} \right]$$

$$\Rightarrow 0.88 - 1 = 0.05 \times \left[1 - \left(\frac{P_2}{P_1} \right)^{1/n} \right]$$

$$\Rightarrow \frac{-0.12}{0.05} = 1 - \left(\frac{P_2}{P_1}\right)^{1/n}$$

$$\Rightarrow -2.4 = 1 - \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

$$\Rightarrow \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} = 3.4$$

$$\Rightarrow \frac{P_2}{P_1} = 3.4^{1.25} = 4.6169$$

$$\Rightarrow P_2 = 4.6169 \times 115 = 530.9435 \text{ kN/m}^2$$

As minimum work input, stage pressure ratios are equal.

$$\therefore \frac{P_3}{P_2} = \frac{P_2}{P_1} = 4.6169$$

$$\Rightarrow P_3 = 4.6169 \times P_2 = 4.6169 \times 530.9435$$

$$\Rightarrow P_3 = 2451.313 \text{ kN/m}^2$$

Therefore, the stage delivery pressures are 530.9435 kN/m² and 2451.313 kN/m²

(a)(ii)

$$T_2 = T_1 \times \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = 298 \times 4.6169^{\frac{0.25}{1.25}} = 404.656 \text{ K}$$

$$IP = 3 \times \frac{n}{n-1} \times \dot{m} \times R \times (T_2 - T_1)$$

$$\Rightarrow IP = 3 \times \frac{1.25}{0.25} \times 0.1333 \times 0.287 \times (404.656 - 298)$$

$$\Rightarrow IP = 61.2052 \text{ kW}$$

Therefore, the indicated power is 61.2052 kW.

(a)(iii)

$$\dot{Q}_{ic \text{ total}} = \dot{Q}_{ic1} + \dot{Q}_{ic2} = 2 \times \dot{Q}_{ic1} = 2 \times \dot{m} \times c_p \times (T_2 - T_{2'})$$

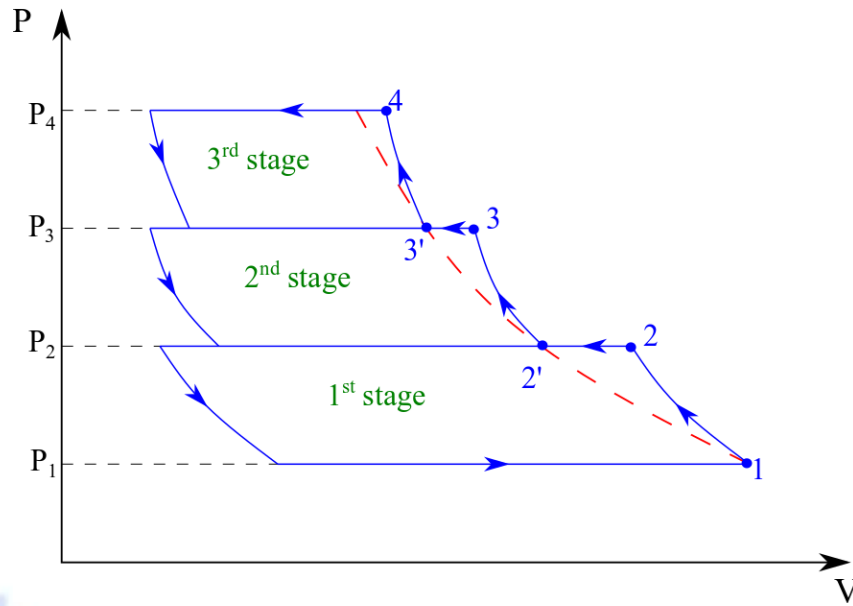
$$\Rightarrow \dot{Q}_{ic \text{ total}} = 2 \times 0.1333 \times 1.005 \times (404.656 - 298)$$

{ \therefore Perfect intercooling $T_{2'} = T_1 = 298 \text{ K}$ }

$$\Rightarrow \dot{Q}_{ic \text{ total}} = 28.577 \text{ kW}$$

Therefore, the total heat removed in the intercoolers is 28.577 kW.

(b)



Q.9

A reducing bend is fitted in a horizontal section of a fresh water system as shown in Fig Q9.

It turns the flow through an angle of 90° anticlockwise to the direction of flow.

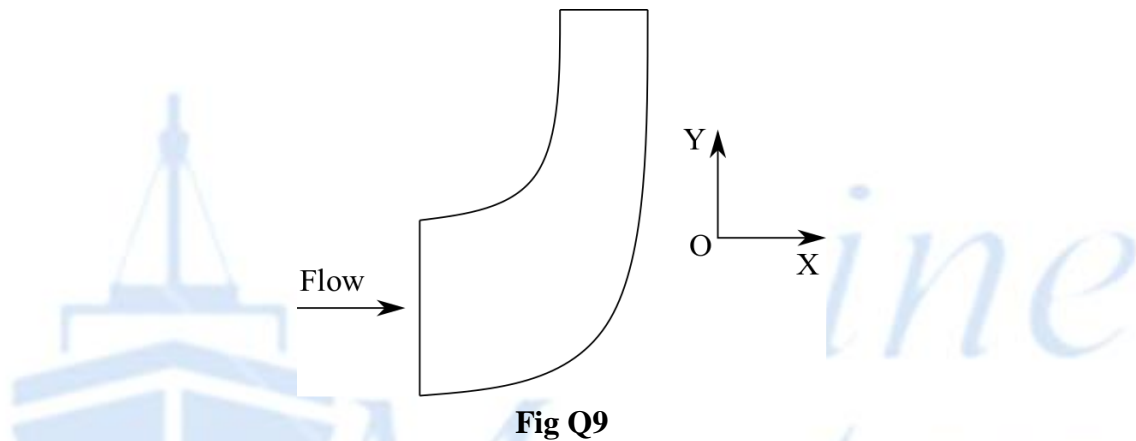
The system pressure and fluid velocity at inlet are 8 bar and 1.5 m/s respectively.

The bend has diameters of 300 mm at inlet and 150 mm at outlet.

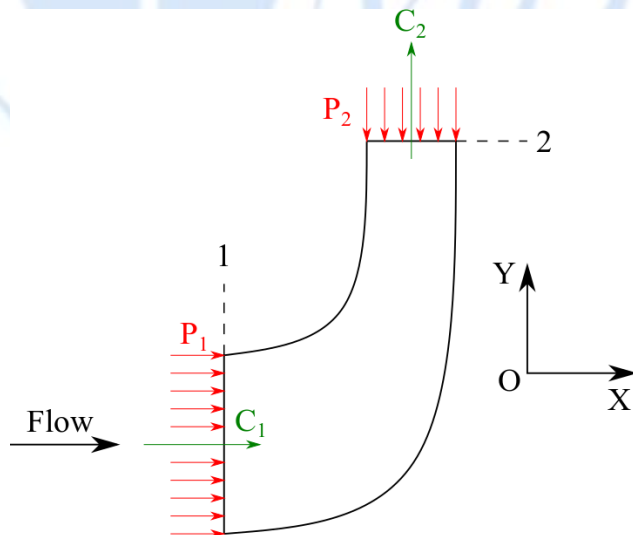
The friction loss in the bend may be ignored.

Calculate EACH of the following:

- the system pressure at the bend outlet; (4)
- the forces acting in the X and Y direction due to the change in diameter; (3)
- the forces acting in the X and Y direction due to the change in momentum; (3)
- the magnitude of the resultant force acting on the bend; (4)
- the direction of the resultant force. (2)



Solution:



As horizontal section, $z_1 = z_2$

$$P_1 = 8 \text{ bar} = 8 \times 10^5 \text{ N/m}^2$$

$$C_1 = 1.5 \text{ m/s}$$

$$D_1 = 300 \text{ mm} = 0.3 \text{ m}, \quad D_2 = 150 \text{ mm} = 0.15 \text{ m}$$

$$\text{As fresh water, } \rho = 1000 \text{ kg/m}^3$$

By continuity equation,

$$A_1 C_1 = A_2 C_2$$

$$\Rightarrow \frac{\pi}{4} \times D_1^2 \times C_1 = \frac{\pi}{4} \times D_2^2 \times C_2$$

$$\Rightarrow C_2 = \frac{D_1^2 \times C_1}{D_2^2} = \frac{0.3^2 \times 1.5}{0.15^2} = 6 \text{ m/s}$$

$$\dot{m} = \rho A_1 C_1 = 1000 \times \frac{\pi}{4} \times 0.3^2 \times 1.5 = 106.02 \text{ kg/s}$$

(a)

By Bernoulli equation,

$$\frac{P_1}{\rho g} + \frac{C_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{C_2^2}{2g} + z_2$$

$$\Rightarrow \frac{P_2}{\rho g} = \frac{P_1}{\rho g} + \frac{C_1^2}{2g} - \frac{C_2^2}{2g}$$

$$\Rightarrow P_2 = P_1 + \rho \times \frac{C_1^2 - C_2^2}{2}$$

$$\Rightarrow P_2 = 8 \times 10^5 + 1000 \times \frac{1.5^2 - 6^2}{2}$$

$$\Rightarrow P_2 = 783125 \text{ N/m}^2$$

Therefore, the system pressure at the bend outlet is 783125 N/m².

(b)

Force acting in X direction due to change in diameter

$$= P_1 A_1 = 8 \times 10^5 \times \frac{\pi}{4} \times 0.3^2 = 56548.66 \text{ N}$$

Force acting in Y direction due to change in diameter

$$= P_2 A_2 = 783125 \times \frac{\pi}{4} \times 0.15^2 = 13838.96 \text{ N}$$

Therefore, the forces acting in the X and Y direction due to change in diameter are

56548.66 N and 13838.96 N respectively.

(c)

Let,

 $F = \text{Force acting on the bend due to flow of water}$ $R = \text{Force acting on the water due to bend}$ $R_x = \text{Component of } R \text{ in } X \text{ direction}$ $R_y = \text{Component of } R \text{ in } Y \text{ direction}$

F and R are equal in magnitude and opposite in direction.

Applying momentum equation in X direction

$$\Sigma F_x = \dot{m}(C_{2x} - C_{1x})$$

$$\Rightarrow P_1 A_1 + R_x = 106.02 \times (0 - C_1)$$

$$\Rightarrow R_x = -106.02 \times C_1 - 56548.66$$

$$\Rightarrow R_x = -106.02 \times 1.5 - 56548.66$$

$$\Rightarrow \mathbf{R_x = -56707.69 N}$$

Applying momentum equation in Y direction

$$\Sigma F_y = \dot{m}(C_{2y} - C_{1y})$$

$$\Rightarrow -P_2 A_2 + R_y = \dot{m}(C_2 - 0)$$

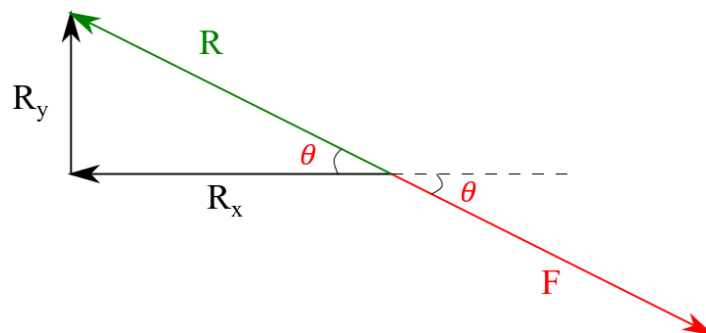
$$\Rightarrow R_y = \dot{m} C_2 + P_2 A_2 = 106.02 \times 6 + 13838.96$$

$$\Rightarrow \mathbf{R_y = 14475.08 N}$$

Therefore, the forces acting in the X and Y direction due to change in momentum are

56707.69 N and 14475.08 N respectively.

(d)



$$F = R = \sqrt{R_x^2 + R_y^2} = \sqrt{56707.69^2 + 14475.08^2} = 58525.98 \text{ N}$$

Therefore, the magnitude of the resultant force acting on the bend is 58525.98 N.

(e)

$$\tan \theta = \frac{14475.08}{56707.69}$$

$$\Rightarrow \theta = 14.32^\circ$$

Therefore, the direction of resultant force is 14.32° clockwise w.r.t. inlet flow.

